

Welcome to Day 2!!!!

- Please complete the Day 1 Reflection Form and place on the back table before we begin the session today.



Algebraic Reasoning Content Academy – Grade 5

Day 2: Multiple Representations
Sami Briceño

Adding It Up: Helping Children Learn Mathematics

- Conceptual Understanding: comprehension of concepts, operations and relations
- Procedural Fluency: skill in flexibly, accurately, efficiently, and appropriately using procedures
- Strategic Competence: ability to formulate, represent and solve problems
- Adaptive Reasoning: capacity for logical thought, reflection, explanation, and justification
- Productive Disposition: view of math as useful, sensible and worthwhile coupled with belief in diligence and personal efficacy



Kilpatrick, J., Swafford, J., Findell, B. (Ed.). (2001). Adding it up: helping children learn mathematics. Washington, DC: National Academy Press.

Problem Solving—Van De Walle

- The problem must begin where the students are
- The problematic or engaging aspect of the problem must be due to the mathematics
- The problem must require justifications and explanations for answers and methods.

Show & Tell Approach—Van De Walle

Listen to a description from Van de Walle's book of outcomes in a teacher directed approach.

Problem Solving—Van De Walle

- Focuses students' attention on ideas and sense making.
- Develops the belief in students that they are capable of doing mathematics and that mathematics makes sense.
- Provides ongoing assessment data.
- Excellent method for attending to a breadth of abilities.
- Develops “mathematical power.”
- IT IS A LOT OF FUN

Provide Hints but Not Solutions —Van De Walle

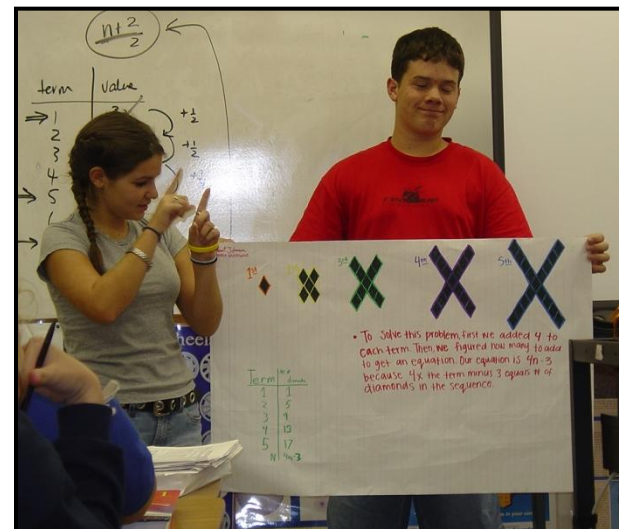
“Always keep in mind that as soon as students sense that you have a method of solving the problems, they will almost certainly stop searching for their own methods because they are convinced that your way must be best.”

Gr 3-5 Teaching Student-Centered Mathematics—Van De Walle pg 21-24

- What can I tell them? Should I tell them anything?
- How will I be able to teach all of the basic skills?
- Why is it okay for a student to “tell” or “explain” but not for me?
- This approach take more time, how will I have time to cover everything?
- Do I need to use a problem-based approach every day?
- Is there any place for drill and practice?
- My textbook is traditional, how can I use it?
- What do I do when a task bombs or students don’t “get it”?

Require students to defend and talk about their solutions because communication deepens understanding.

When did the mathematics make sense for you?



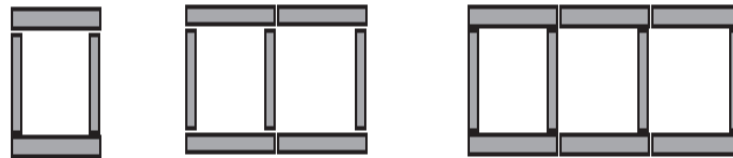
#1 Answer:

When I started teaching it!!!

Day 2 Learning Outcomes

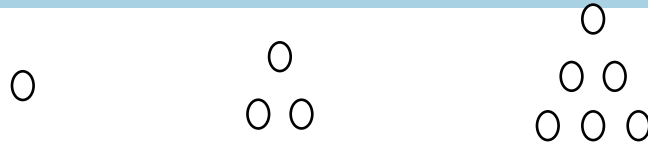
- Use both real-life and abstract representations to solve problems
- Investigate what can be learned from analyzing students' work
- Introduce the value of multiple representations: table, verbal/pictorial, equation, graph
- Translate and make connections among multiple representations, including technology
- Solve, analyze, and discuss 3rd grade algebraic reasoning tasks, connection to MS & HS Algebra Tasks

Growing Bigger



- Predict how many popsicle sticks will be needed to create Figure 15
- Explain your reasoning for your prediction
- Create a table to organize the data for the figure number and the number of popsicle sticks needed to create each figure.
- Extend the pattern to check your prediction for the number of popsicle sticks needed to create Figure 15. Were you correct in your original prediction? If not, figure out why your prediction was incorrect and explain the reasoning of the correct pattern.

Growing Bigger



- How many counters did you use for each triangle?
- What happens between the second and the third triangles?
- Describe any patterns you notice in the triangles
- How many counters would it take to make the sixth triangle? How do you know?
- Describe how you can find the number of counters in the tenth triangle.

Tile Patterns



Tiling around a Fountain

- **SCENARIO:** You are working on a landscaping crew for the county parks department this summer.
- To avoid the mud that surrounded the park fountains last summer, your crew is planning to put a border of tiles around each of the square fountains in the park. The border tiles each measure 1-foot on each side. Your foreman shows you this diagram for the smallest fountain. You notice that a fountain that has a base of 1 square foot will require 8 border tiles.
- Using this pattern, how many tiles will be needed for different size square fountains?



Tiling around a Fountain

Problem 1: If a square fountain has sides of length s feet, how many tiles are needed to form the border?

- Using grid paper draw a diagram of the designs for the border of fountains with side lengths of 2, 3, 4, 6 and 10 feet. Record your results in a table.
- Write an equation for the number of tiles, N , needed to form a border for a square fountain with side length of S feet.
- Generate as many equations as you can for this relationship.
 - Are the equations the same?
 - How can you convince someone that your expressions for the number of tiles needed are equivalent?



Group Poster

Tiling around a Fountain

- Show all the expressions generated by your group
- Draw Fountain with Side Length 3, ***explicitly connect all of your expressions to this diagram***



Gallery Walk

Tiling around a Fountain

- One person from each group “mans” the group’s poster to answer questions.
- Rest of the group members view other posters. Look for:
 - The most common representations
 - The most unique representations



Re-Tiling

Tiling around a Fountain

Follow-up Problem: Make a table and a graph for each equation you found in problem 1.

- Do the tables and graphs show that the expressions and equations are equivalent?
- Is this relationship between s and n a straight line, or a curve?



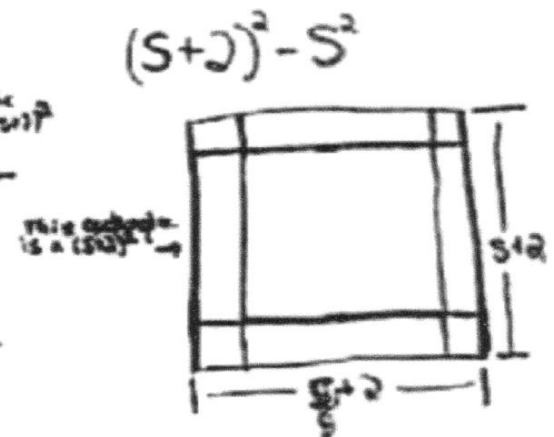
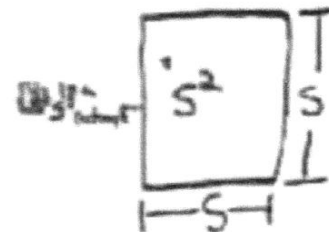
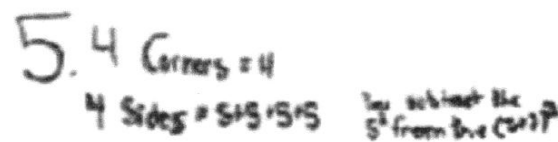
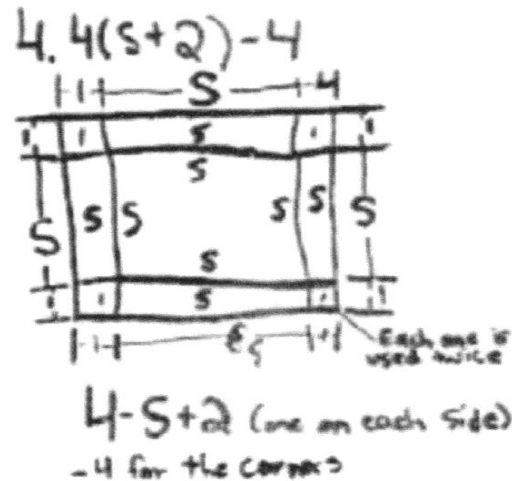
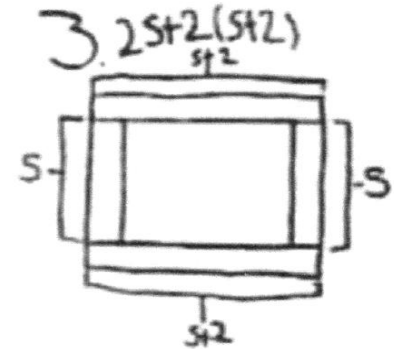
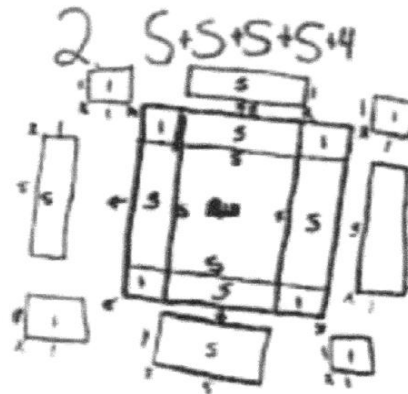
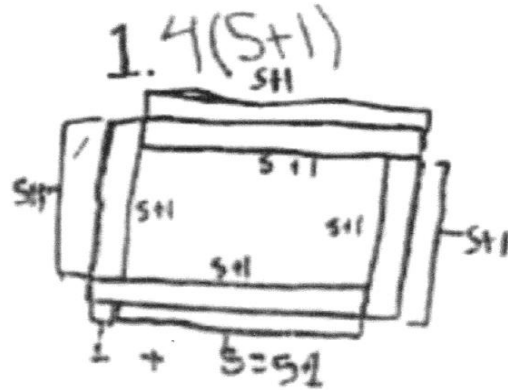
Collection of Expressions

Tiling around a Fountain

- Make a collective list of all the expressions that were generated in the groups.

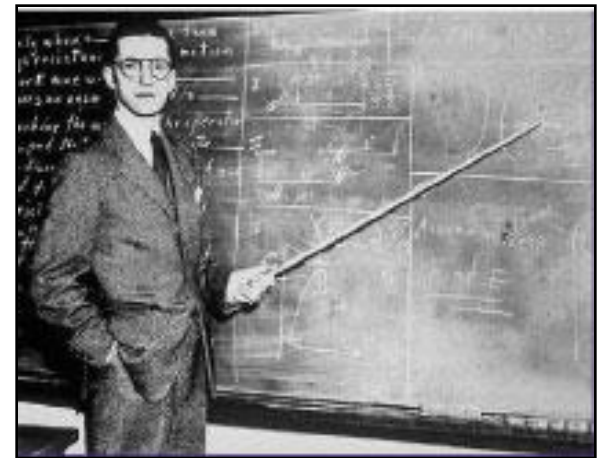


Tiling around a Fountain



Looking through Teacher Lenses

- How would you characterize the level of this task: High or low cognitive demand?
- What mathematical ideas are embedded in the task?
- What makes this worthwhile mathematics?



Why do we want kids to experience Pattern Tasks?

- Understanding variables
- Connecting between representations
- Progressing on standards
- Using context for working on functions
- Starting the problem at multiple entry points
- Finding and comparing multiple solutions



Broken Gumball Machine



Something was wrong with the gumball machine at the shopping mall. If you put in one quarter and then turned the knob, you got 5 gumballs. If you put two quarters and then turned the knob, you got 8 gumballs. If you put in three quarters and then turned the knob, you got 11 gumballs. If you put in four quarters and then turned the knob, you got 14 gumballs. If John put 5 quarters in the machine and then turned the knob, how many gumballs would he get if this pattern continued? What if Joyce puts in 10 quarters and then turns the knob, how many gumballs will she get? Please organize your work so someone else reading it will understand it.

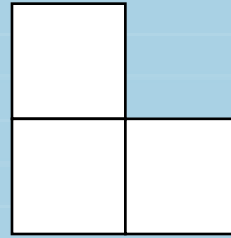
Broken Gumball Machine



Bonus: If you put in \$5.00 in quarters and then turned the knob, how many gumballs would you get?

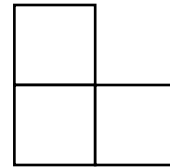
Extension: Write a rule for finding how many gumballs you will get if you put any number of quarters in the gumball machine.

L's for Linda

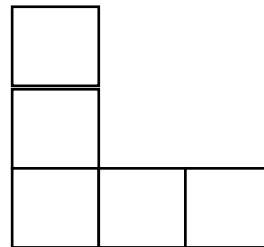


Linda likes to do art projects. She also likes the letter L because it is the first letter of her name. Linda decided to decorate her bedroom with L's made with colored plastic tiles.

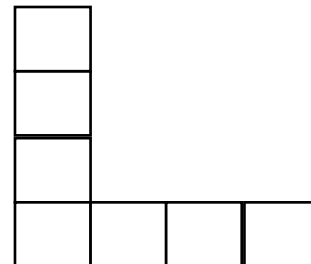
The first L took 3 tiles to make:



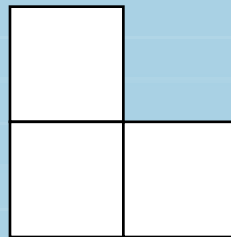
The second L took 5 tiles to make:



The third L took 7 tiles to make:

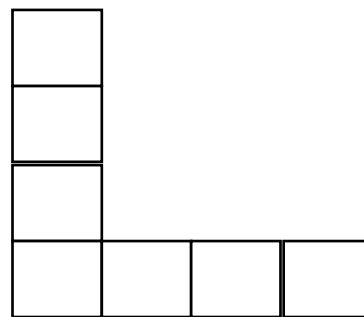
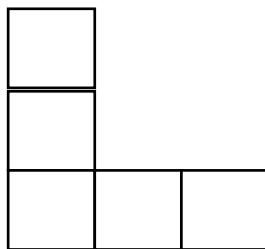
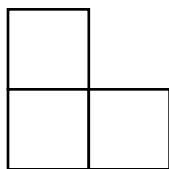


L's for Linda

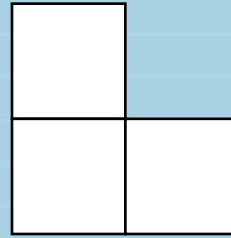


Linda didn't know how many L's she would make, but she was sure she would make at least 10. She might even make 50!

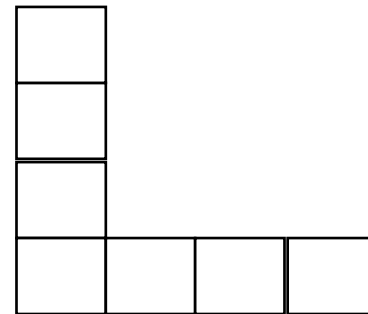
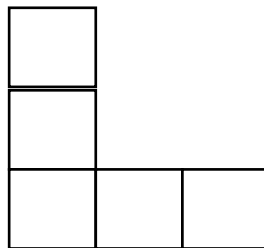
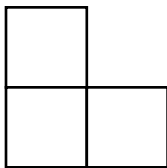
If Linda continues to make L's in the same pattern established above, how many tiles would Linda need to make the tenth L? How many tiles would she need to make the fifteenth L? Show your math thinking.



L's for Linda



Linda went to the hobby store and discovered that a bag of 50 colored tiles cost \$1.30 (that included tax). She also noticed that a bag of 65 colored tiles cost \$1.50 (that also included tax). If Linda makes just the fifteenth L, what bags of tiles should she buy that will give her the best deal? Show your math thinking.



Analyzing Student Work

- Examine each students' response. What does it suggest the student understands? Does not understand? Why?
 - Which response do you think shows the greatest understanding? Why?
 - Which response do you think shows the least understanding? Why?
- Rank your group's student responses from weakest to strongest using a 1, 2, 3, and 4 ranking. 1 being weakest understanding.

Analyzing Student Work

- What are the characteristics of a response that indicates understanding?
- What are the characteristics of a response that indicates a lack of understanding?

U.S. Shirts

Using Tables, Graphs, and Equations

This past summer you were hired to work at a custom T-shirt shop, U.S. Shirts. One of your responsibilities is to find the total cost of customers' orders. The shop charges \$8 per shirt with a one-time set-up fee of \$15.

1. What is the total cost of an order for 10 shirts?
2. What is the total cost of an order for 100 shirts?
3. Explain how you found the total costs.
4. How many T-shirts can a customer buy for \$60?
5. How many T-shirts can a customer buy for \$250?
6. Explain how you found the number of shirts that can be purchased.



U.S. Shirts

Using Tables, Graphs, and Equations

Make a table of values for the problem situation.

Labels	Number of Shirts Ordered	Total cost
	Shirts	\$
Unit		



U.S. Shirts

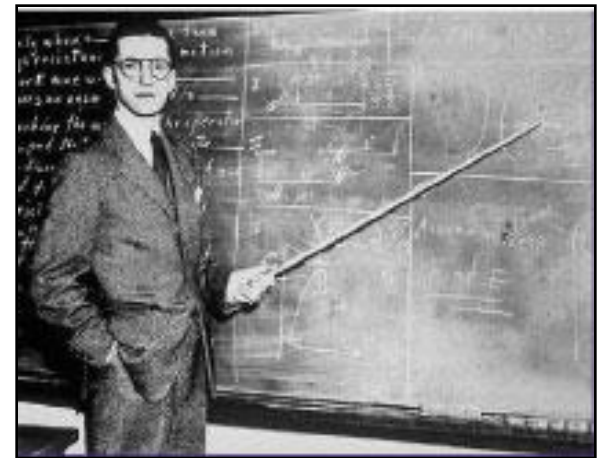
Using Tables, Graphs, and Equations

- Write an algebraic equation for the problem situation.
- In this lesson, you have represented the problem situation in four different ways: as a sentence, as a table, as a graph, and as an equation. Explain the advantages and disadvantages of each representation.



Looking through Teacher Lenses

- How would you characterize the level of this task: High or low cognitive demand?
- What mathematical ideas are embedded in the task?
- What makes this worthwhile mathematics?



Final Thoughts

“And once I had a teacher who understood. He brought with him the beauty of mathematics. He made me create it for myself. He gave me nothing, and it was more than any other teacher has ever dared to give me.”

Cochran (1991)